Faster Inference of Cell Complexes from Flows via Matrix Factorization

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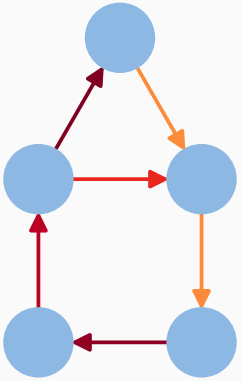
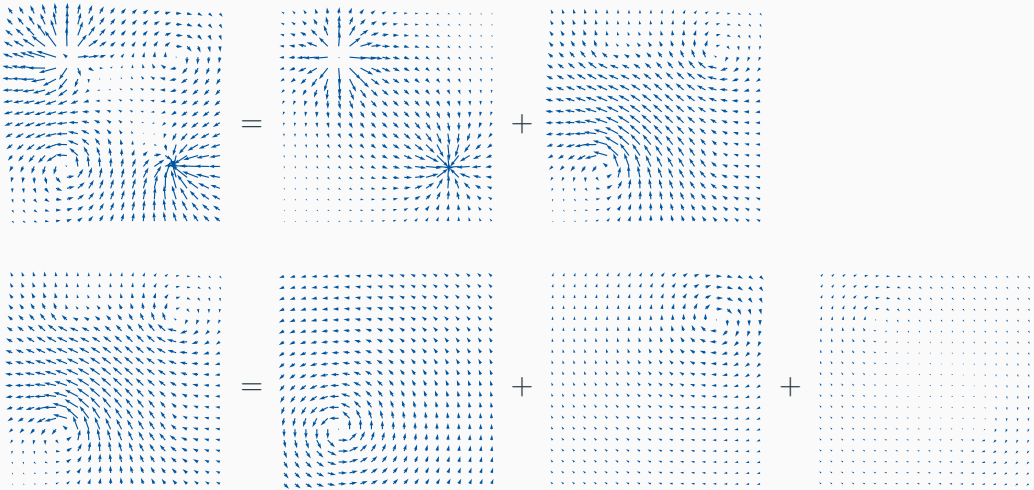
Motivation: Edge Flows

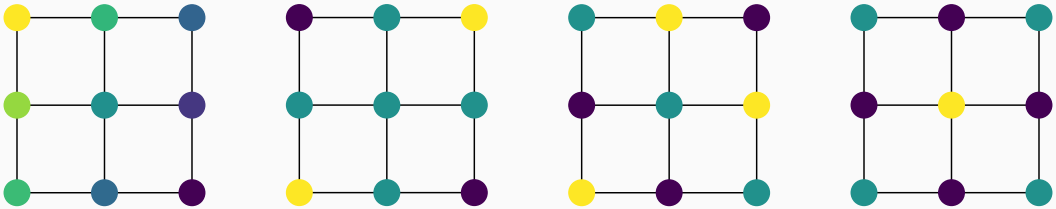


Intuition: Continuous case



Background: Graph Signal Processing

Graph Laplacian $L_0 = D - A$ is used as a shift operator.

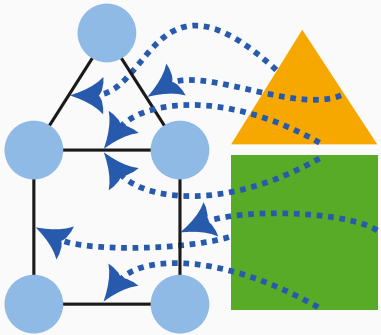


$\lambda_1 = 1$ $\lambda_3 = 2$ $\lambda_6 = 4$ $\lambda_8 = 6$

Low-pass / high-pass filters based on eigendecomposition of L_0.

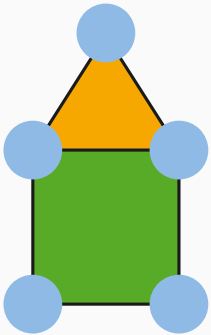
- image denoising
- discrete Fourier transform is special case

Background: Cell Complexes



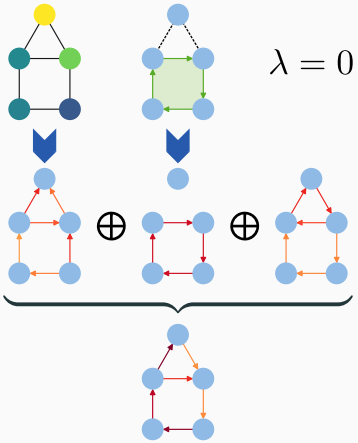
1-dim:
Graph

Polygons
are glued
to edges



2-dim
Cell Complex

Shift Operator: Hodge Laplacian

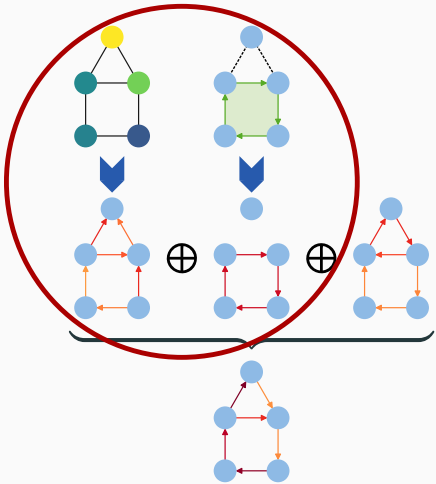


Problem Statement: Sparse Approx.

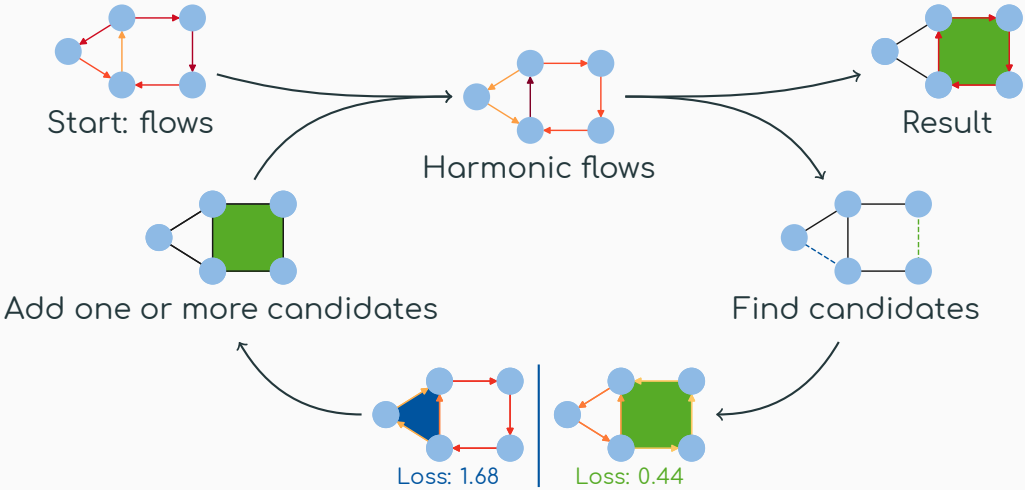
Given edge flows F on a graph:

$$\min \left|\left|\mathrm{harm}_{L_1}(F)\right|\right|_F \quad \text{s.t.} \quad \mathrm{rk}\, L_1 \leq k$$

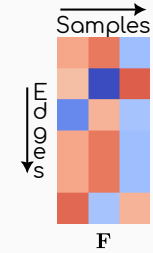
- Boundaries of 2-cells are dictionary entries
- Sparse representation shows significant signal components
- Filtering over non-harmonic eigenspace of Hodge Laplacian



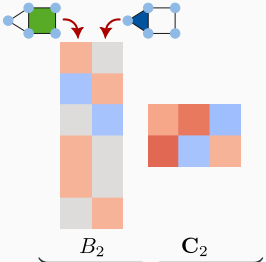
Iterative Cell Inference



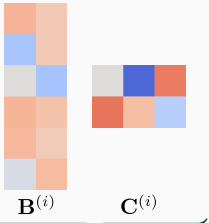
Matrix-Factorization-Based Heuristic



harmonic flows

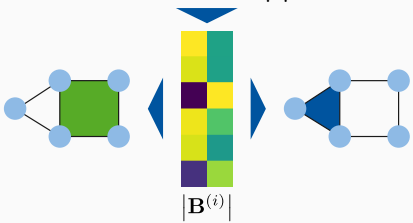


generating model

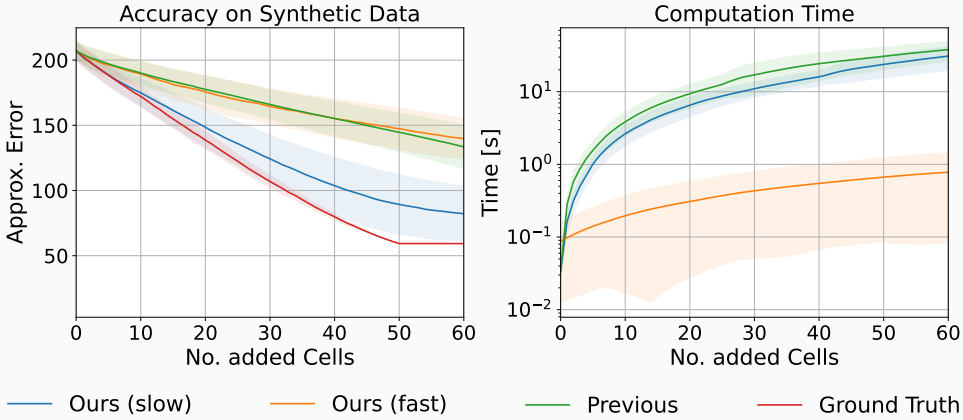


continuous approximation

Candidates from edges with greatest values in each colum:

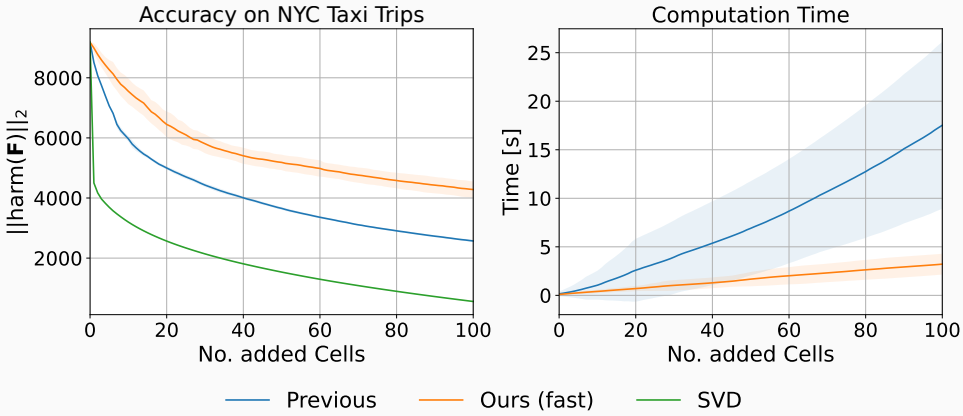


Experiments: Synthetic Data



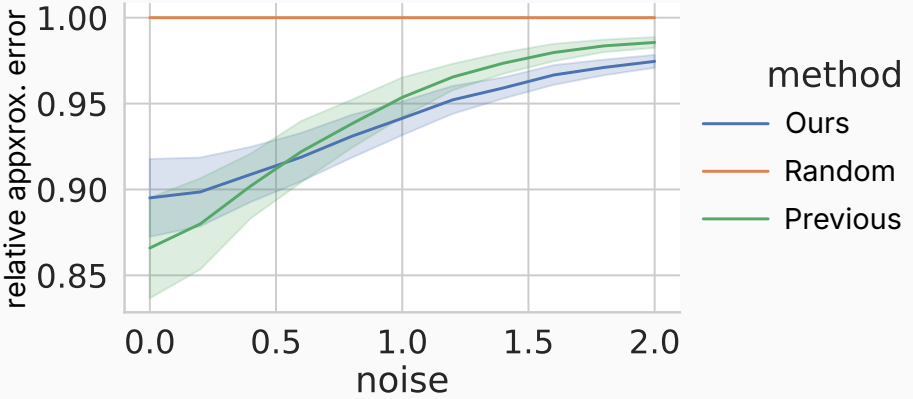
$G(n, \rho)$-graph with $n = 40$, $\rho = 0.9$ and 50 2-cells; 64 flows, gaussian noise with $\sigma = 0.3$. Inference using ICA (optimal).

Experiments: NYC Taxi Trips



128 flows. Inference using SVD (optimal)

Experiments: Noise



$G(n, \rho)$-graph with $n = 40$, $\rho = 0.9$ and 80 2-cells, 64 flows. 40 2-cells inferred.
Inference using SVD to simulate real-world results.

Conclusion

- Generating model important
  - Model match: Clear improvement
    - Much faster with same accuracy
    - Much more accurate with same speed
  - Model Mismatch (real-world): Tradeoff
    - Faster but less accurate
- Noise: Matrix factorization may be useful as preprocessing step
- *Don't be Afraid of Cell Complexes!*
  CCs and topological signal processing
  see `hoppe.io` or `arXiv:2506.09726`

`arXiv:2508.21372`