Don’t be Afraid of Cell Complexes!

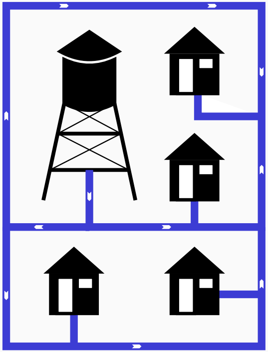
An Introduction to Cell Complexes and Topological Signal Processing from an Applied Perspective

Josef Hoppe

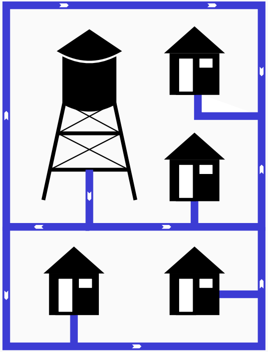
June 2026

Graph Signal Processing Workshop 2026

Computational Network Science
RWTH AACHEN UNIVERSITY



GSP traditionally focuses on node signals, but what about signals on edges?



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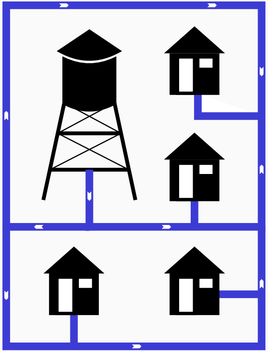
- Flow in water distribution networks



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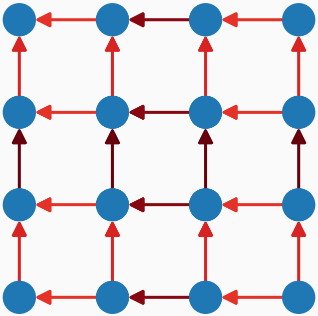
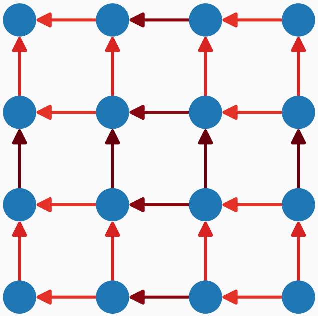
- Flow in water distribution networks
- Traffic flow
- Information flow
- Power transfer in grids



Smooth Flow
(Global Trend)



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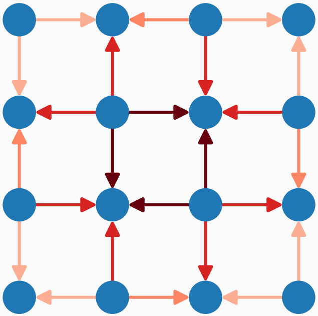
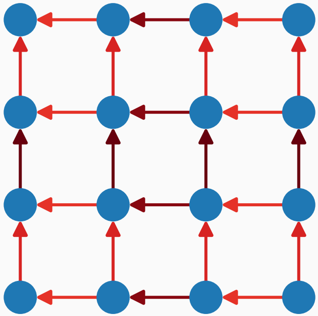


Noisy Flow
(Local Fluctuations)



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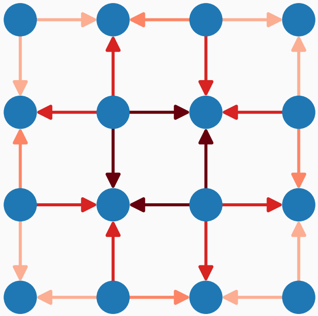
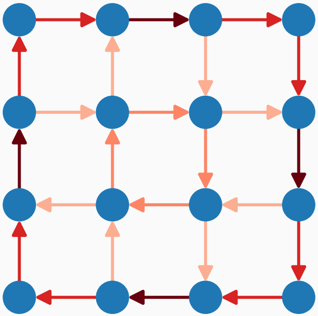


Noisy Flow
(Local Fluctuations)

Smoothness via Node in-/ outflow

Processing Edge Signals II



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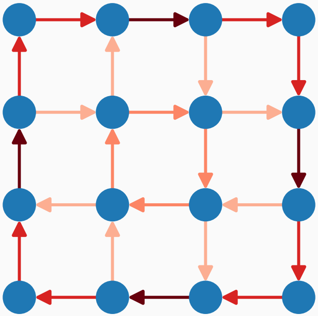

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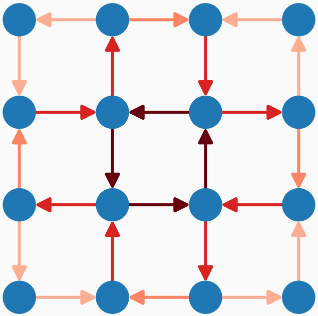

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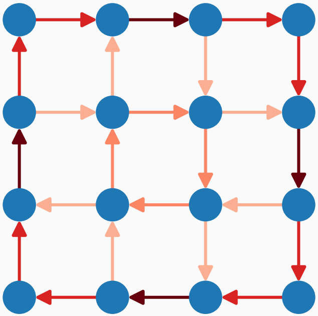

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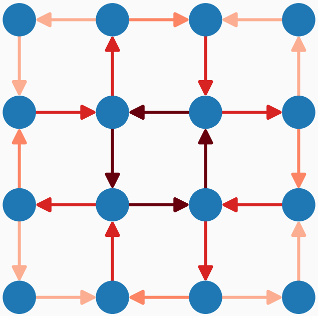
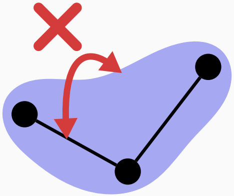


Noisy Flow
(Local Fluctuations)

Nodes have net 0 in-/ outflow, Smoothness via Circulation

What kind of higher-order structure do we need?

Why are Cell Complexes the right tool?



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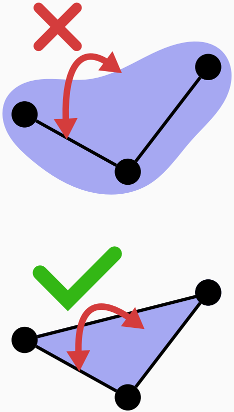
- Hypergraphs lack structure



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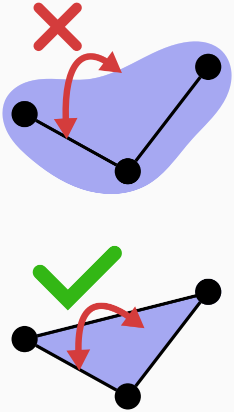
- Hypergraphs lack structure
- Simplicial Complexes



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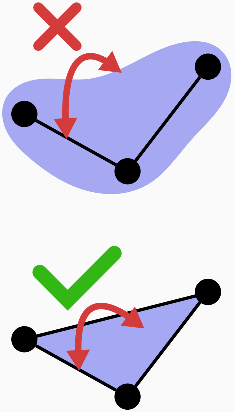

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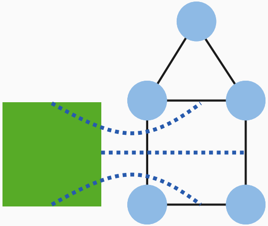
$\Rightarrow$ **Cell Complexes!**



Goal and Outline

If you know:

- Graphs,
- Linear Algebra, and
- Basic Combinatorics

this talk is for you!

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Goal and Outline

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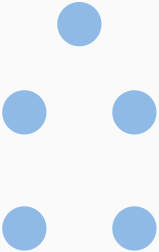
I will introduce:

1. (Abstract) Cell Complexes
2. Orientation and Signals
3. Topological Signal Processing

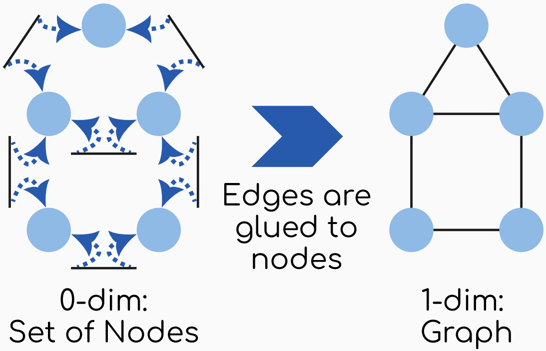
(Abstract) Cell Complexes

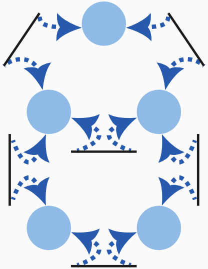
Constructing Cell Complexes



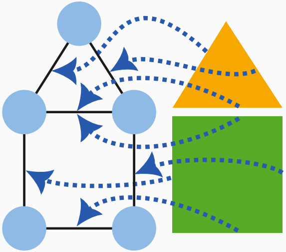
0-dim:
Set of Nodes

Constructing Cell Complexes





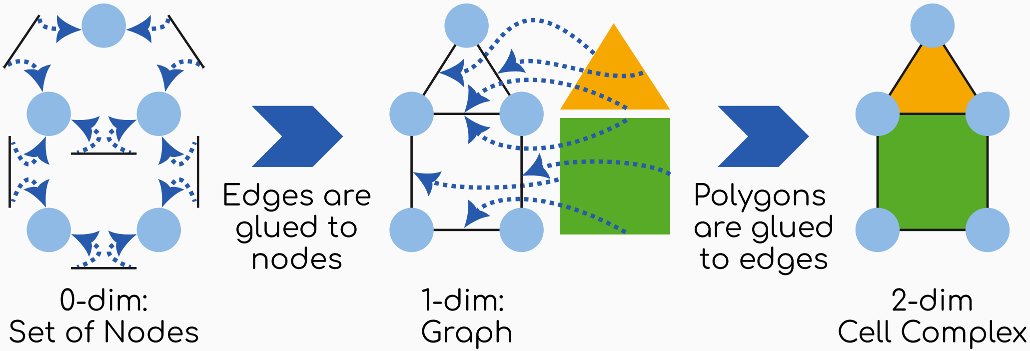
Edges are glued to nodes



0-dim: Set of Nodes

1-dim: Graph

Constructing Cell Complexes



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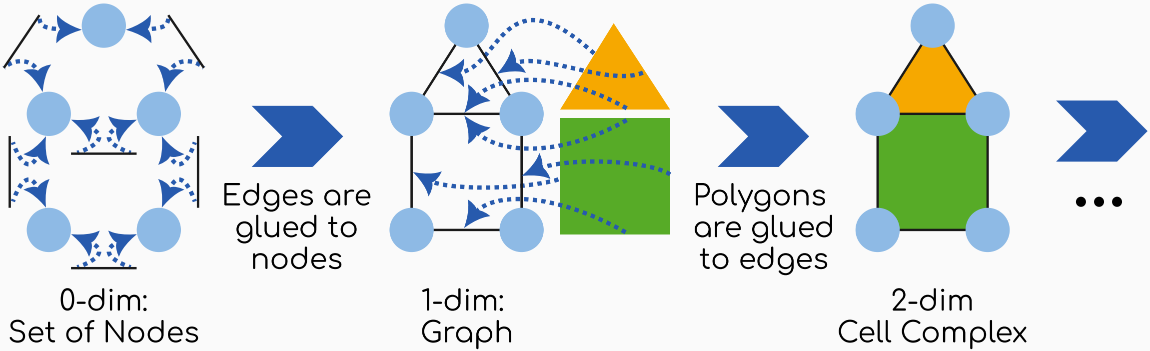
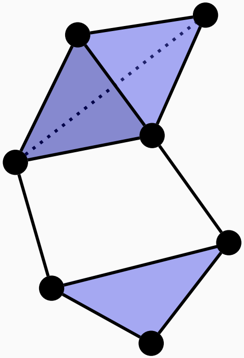
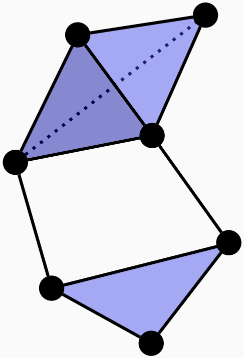
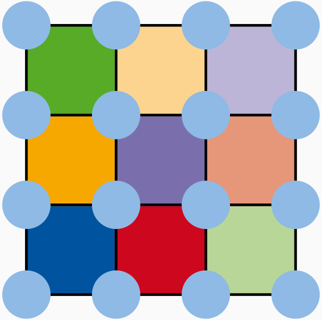


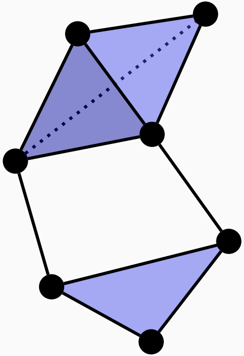


Simplicial Complex

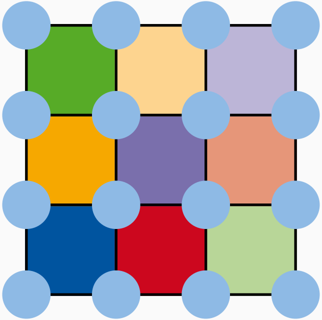

Simplicial Complex


Cubical Complex

How to find Cell Complexes



Simplicial Complex



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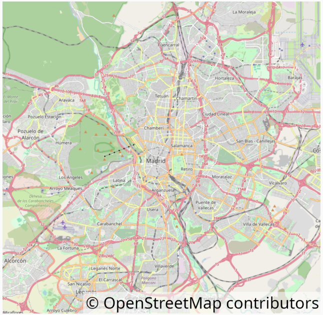
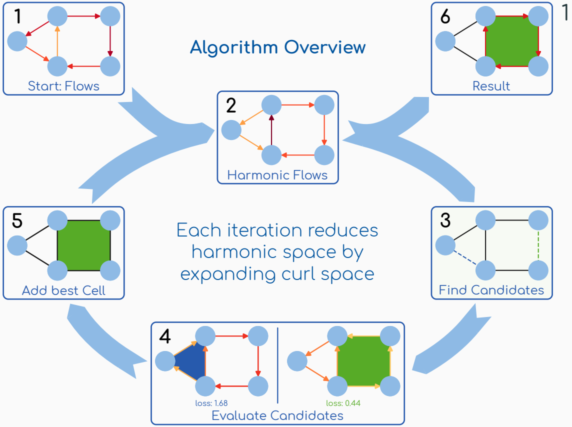


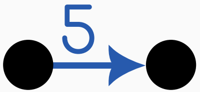
Physical (Geometric) Networks

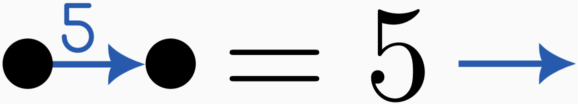


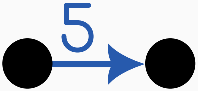
[1] Hoppe and Schaub. "Representing Edge Flows on Graphs via Sparse Cell Complexes." LoG 2023.

Orientation and Signals

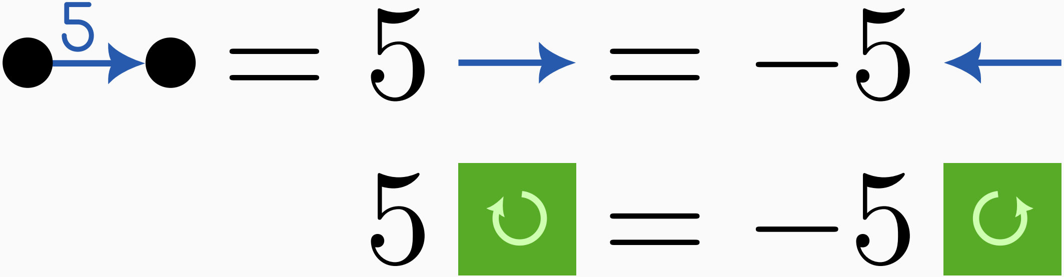
Orientation




5
= 5

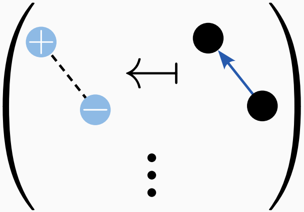


$$= 5 \rightarrow = -5 \leftarrow$$


5
= 5
= −5
5
= −5

Boundaries: Relating Cells of Different Dimensions

$$B_1$$



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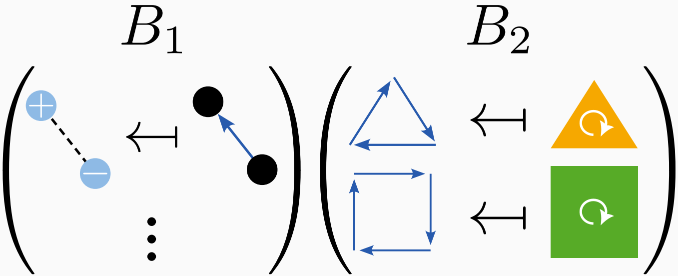
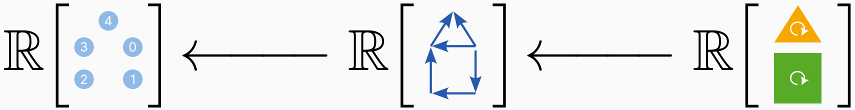
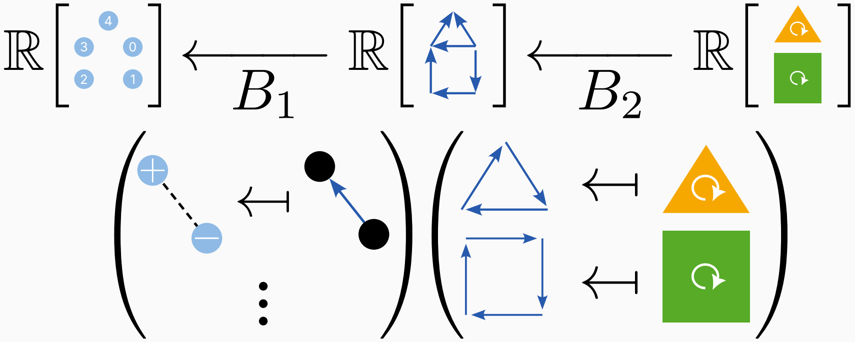


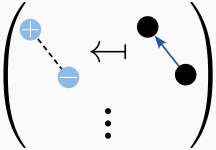
Signal Spaces on Cell Complexes

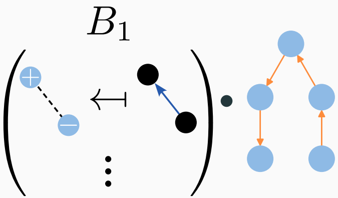


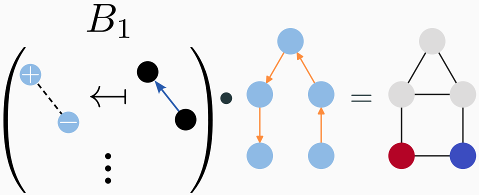

ℝ
4
3
0
2
1
B_1
ℝ
B_2
ℝ

Boundaries: Usage & Interpretation

$$B_1$$




B_1


B_1

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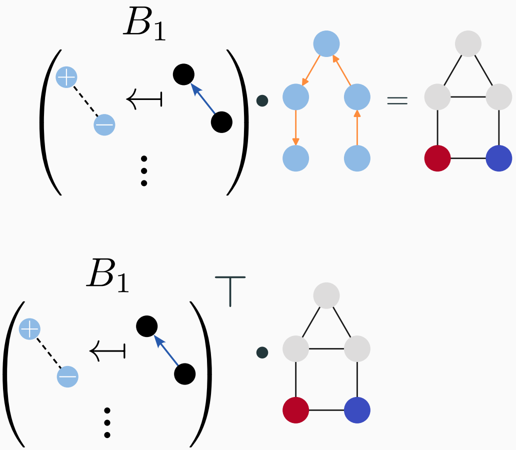

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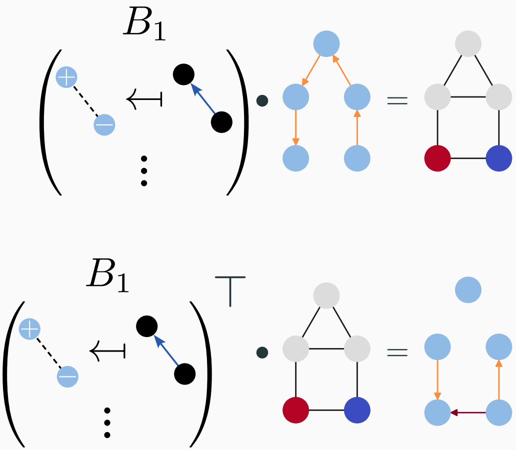
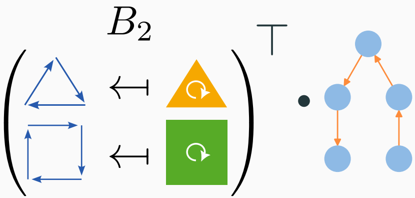
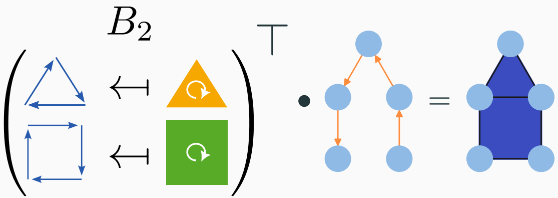



B_2

Boundaries: Usage & Interpretation



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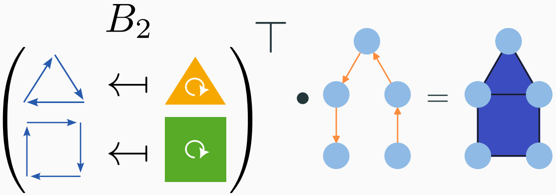

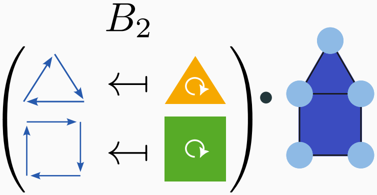

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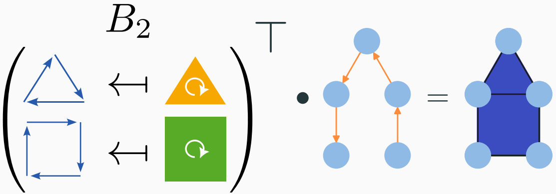

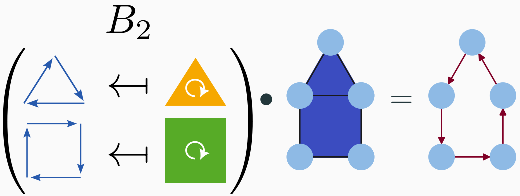

Topological Signal Processing

- Generalization of Graph Signal Processing

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Hodge Laplacian: $L_1 = B_1^\top B_1 + B_2 B_2^\top$

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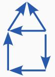
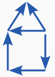
$L_1 : \mathbb{R}\Big[$  $\Big]$ $\qquad \mathbb{R}\Big[$  $\Big]$

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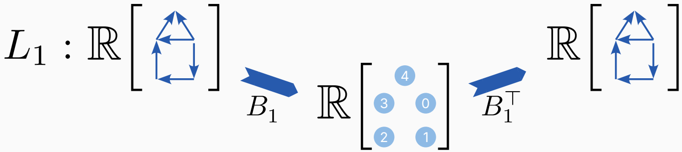

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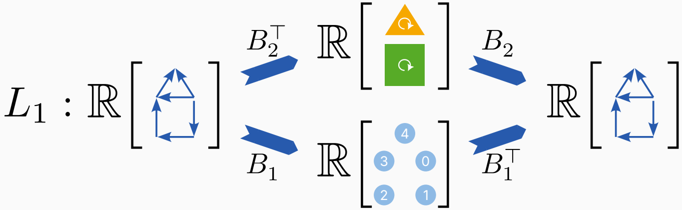

- Generalization of Graph Signal Processing
- Hodge Decomposition

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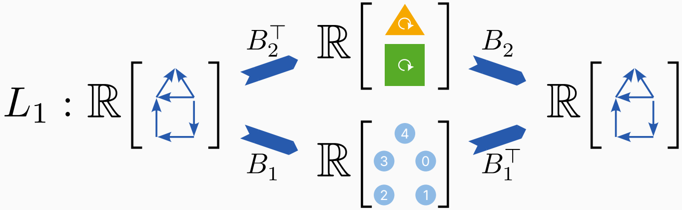

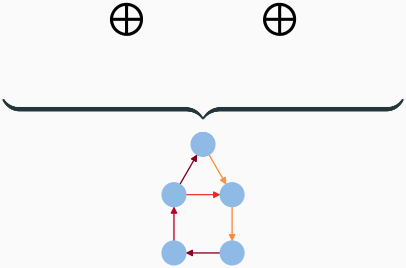

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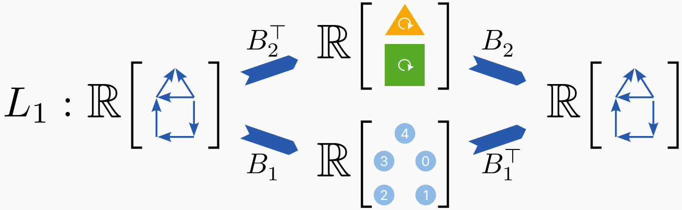

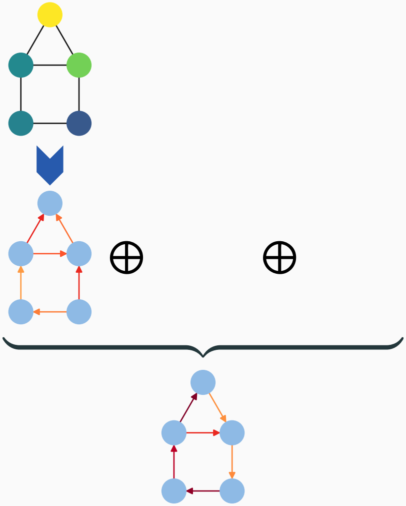

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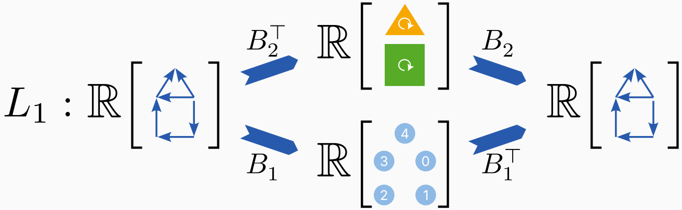

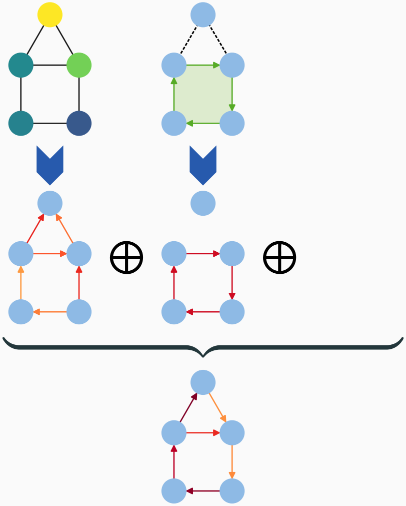

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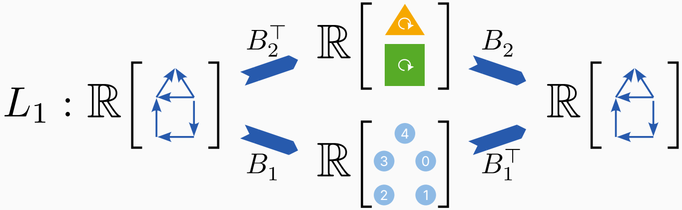

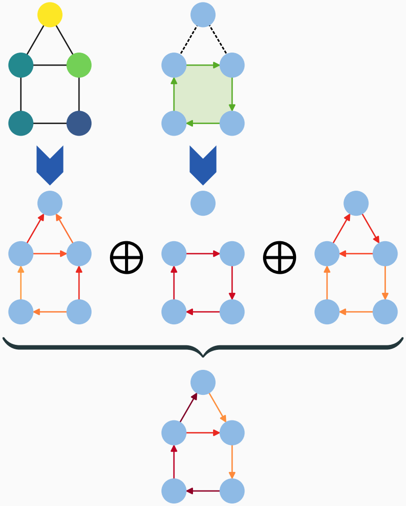

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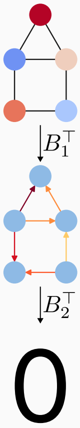
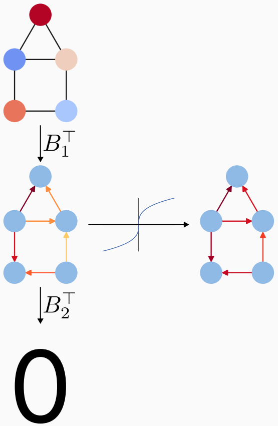
- Node data cannot be projected onto 2-cells



[4] Santoro et al. "From Nodes to Edges: Edge-Based Laplacians for Brain Signal Processing." EUSIPCO 2025.
[5] Liu et al. "Kalman Filtering on Cell Complexes." arXiv preprint arXiv:2605.15955 (2026).

Beyond Edge Signals: Non-linear Signal Processing

- Applying the boundary twice maps every signal to 0.
- Node data cannot be projected onto 2-cells
- Nonlinear processes or element-wise non-linear activation functions can overcome this



[4] Santoro et al. "From Nodes to Edges: Edge-Based Laplacians for Brain Signal Processing." EUSIPCO 2025.

[5] Liu et al. "Kalman Filtering on Cell Complexes." arXiv preprint arXiv:2605.15955 (2026).

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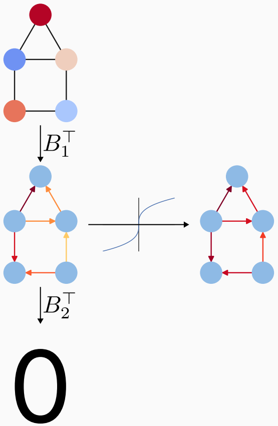

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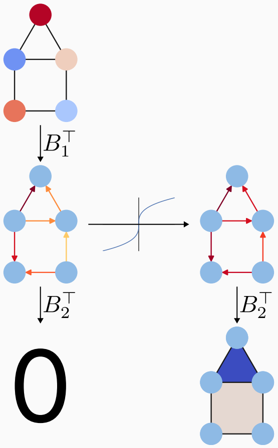
- For example: sine[2], tanh, $\sqrt[3]{x}$, PDEs[3]



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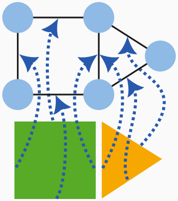
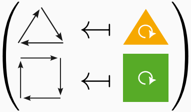
Conclusion



Cell Complex



Boundary Map

Cell Complexes add Structure to Graphs

- Geometric Intuition
- Built upon Boundary Map

Topological SP Applications

- Hodge Decomposition
- Linear TSP / Filter Learning

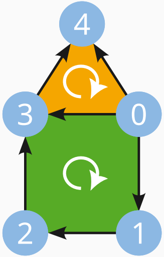
Some Current Research Areas

- Combinatorial Complexity → Heuristics
- Non-linear Dynamics and SP
- Inference of (higher-dimensional) Cells

`hoppe.io`
Personal
Website
Full Paper

Josef Hoppe, Vincent P. Grande, Michael T. Schaub. "Don't be Afraid of Cell Complexes! An Introduction from an Applied Perspective". Preprint, arXiv:2506.09726

Boundary Maps: Example



Cell Complex

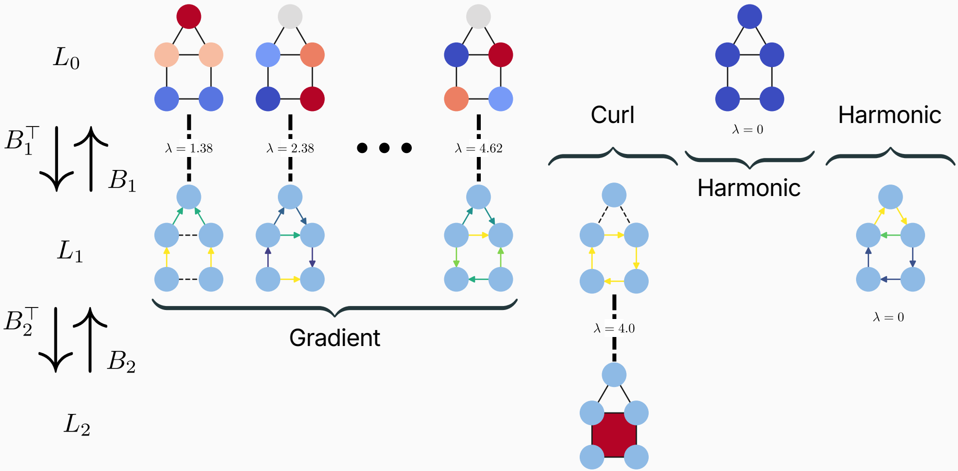
	$0 \to 1$	$0 \to 3$	$0 \to 4$	$1 \to 2$	$2 \to 3$	$3 \to 4$
⓪	$-\mathbf{1}$	$-\mathbf{1}$	$-\mathbf{1}$	0	0	0
①	$\mathbf{1}$	0	0	$-\mathbf{1}$	0	0
②	0	0	0	$\mathbf{1}$	$-\mathbf{1}$	0
③	0	$\mathbf{1}$	0	0	$\mathbf{1}$	$-\mathbf{1}$
④	0	0	$\mathbf{1}$	0	0	$\mathbf{1}$

B_1

		
$0 \to 1$	0	$\mathbf{1}$
$0 \to 3$	$\mathbf{1}$	$-\mathbf{1}$
$0 \to 4$	$-\mathbf{1}$	0
$1 \to 2$	0	$\mathbf{1}$
$2 \to 3$	0	$\mathbf{1}$
$3 \to 4$	$\mathbf{1}$	0

B_2

Connected Signal Spaces



Frequency Filters

Eigendecomposition:

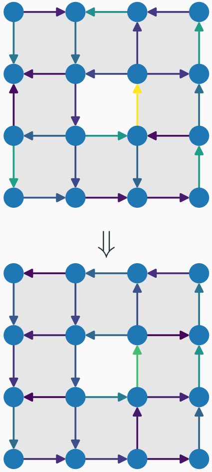
$$L_k = U \Lambda U^T = U \operatorname{diag}(\lambda_0, \lambda_1, ..., \lambda_n) U^\top$$

Spectral Filter:

$$H = U \operatorname{diag}(h(\lambda_0), h(\lambda_1), ..., h(\lambda_n)) U^\top$$

Learnable h, e.g. for time series prediction[4]:

$$h(\lambda) = a_0 + a_1 \lambda + a_2 \lambda^2 + ...$$



[1] Canbolat et al. "Cellular Autoregressive Higher-Order Models." EUSIPCO 2025.