

Introduction to Abstract Cell Complexes and Topological Signal Processing

A Network Science Perspective

Josef Hoppe

November 29, 2024

Learning on Graphs — Paris Meetup



Computational
Network Science

RWTHAACHEN
UNIVERSITY

Cell Complex Neural Network

Thomas Hoff
Department of Electrical and Computer Science
Texas A&M University
hoff@tamu.edu

Christo Pantazis
Department of Computer Science & Engineering
University of South Florida
cpantazi@usf.edu

Abstract

Cell complexes are topological spaces constructed from simple cells. They generalize graphs, simplicial complexes, and simplicial sets.

Weisfelder and Lehman Go Cellular: CC

Christian Bester
University of Cambridge
c.b20@cam.ac.uk

Thomas Hoff
Texas A&M University
hoff@tamu.edu

Christo Pantazis
University of Cambridge
cpantazi@usf.edu

Abstract

NEURAL PROCESSING ON FIBER CCs

T. Mitchell-Anderson¹, Thomas P. Dandekar¹, Florian Frommer², Mohit S. Vaidya³

¹Department of Electrical and Computer Engineering, RWTH Aachen University, Germany

²Department of Computer Science, RWTH Aachen University, Germany

³Department of Computer Science, RWTH Aachen University, Germany

Abstract

We introduce a new class of fiber cell complexes (FCCs) that are suitable for topological data analysis. We study the homology and cohomology of these complexes and show that they admit an efficient algorithm for computing their homology. We also study the relationship between the homology of the fiber and the homology of the total space. Our results show that the homology of the total space can be computed from the homology of the fiber and the homology of the base space.

Signal processing on higher-order networks: Lixiv' on the beyond

Mohit S. Vaidya¹, No Zhan², Jean-Denis Sely², Y. Mitchell Anderson³

¹Department of Computer Science, RWTH Aachen University, Germany

²Department of Electrical and Computer Engineering, RWTH Aachen University, Germany

³Department of Computer Science, RWTH Aachen University, Germany

Abstract

Higher-order networks (HONs) are a natural generalization of graphs and hypergraphs. They are used to model complex systems with higher-order interactions. In this paper, we study the signal processing on HONs. We show that the homology and cohomology of HONs can be computed from the homology and cohomology of their constituent graphs and hypergraphs.

Topological Signal Processing Over Cell Complexes

Stefania Sarracino¹, Saverio Morabito² and Sergio Barbarossa³

¹Department of Electrical and Computer Engineering, RWTH Aachen University, Germany

²Department of Computer Science, RWTH Aachen University, Germany

³Department of Electrical and Computer Engineering, RWTH Aachen University, Germany

Abstract

Topological signal processing (TSP) is a natural generalization of graph-based signal processing. It is used to model complex systems with higher-order interactions. In this paper, we study the topological signal processing over cell complexes. We show that the homology and cohomology of cell complexes can be computed from the homology and cohomology of their constituent graphs and hypergraphs.

Representing Edge Flows on Graphs via Sparse

José Riquelme¹, Michael T. Schaub²

¹Department of Electrical and Computer Engineering, RWTH Aachen University, Germany

²Department of Computer Science, RWTH Aachen University, Germany

Abstract

Edge flows on graphs are a natural generalization of graph-based signal processing. They are used to model complex systems with higher-order interactions. In this paper, we study the representation of edge flows on graphs via sparse matrices. We show that the homology and cohomology of graphs can be computed from the homology and cohomology of their constituent graphs and hypergraphs.

NEURAL PROCESSING ON CELL COMPLEXES

T. Mitchell-Anderson¹, Michael T. Schaub², Mohit Vaidya³

¹Department of Electrical and Computer Engineering, RWTH Aachen University, Germany

²Department of Computer Science, RWTH Aachen University, Germany

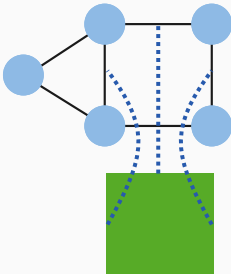
³Department of Mathematics and Computer Science, Texas A&M University, USA

Abstract

Cell complexes are a natural generalization of graphs and hypergraphs. They are used to model complex systems with higher-order interactions. In this paper, we study the neural processing on cell complexes. We show that the homology and cohomology of cell complexes can be computed from the homology and cohomology of their constituent graphs and hypergraphs.

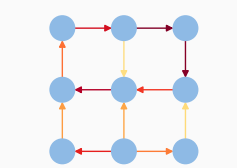


Fig. 1. Complex cell complexes with connections on the edges and vertices (shown in red) and faces (shown in blue). The complex is a 2D simplicial complex with 6 vertices, 9 edges, and 4 faces.

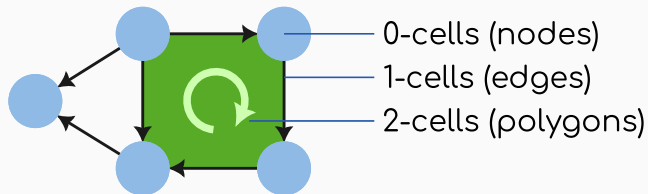


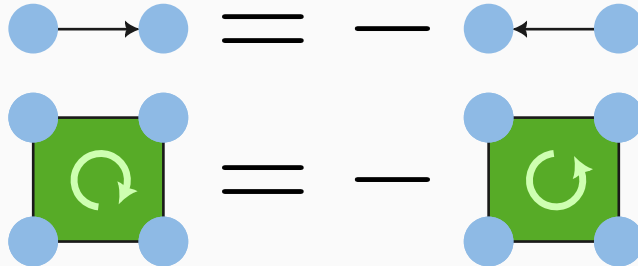
- Higher-Order Structure increases Expressiveness
- CCs enable Signal Processing over Edge Data

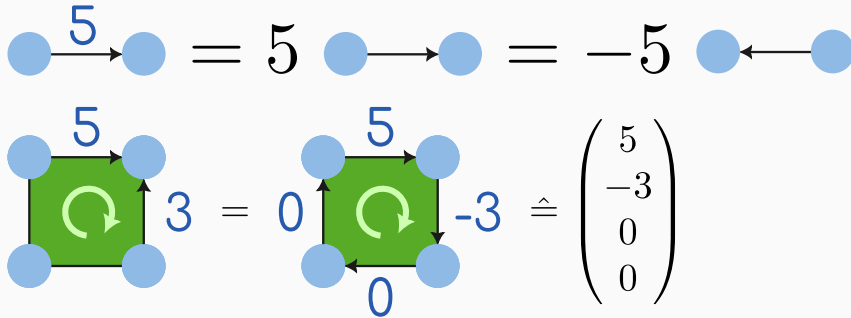
Today: Provide Intuitive Understanding

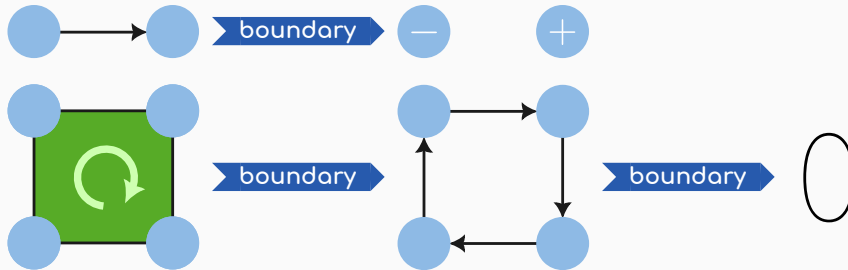


Abstract Cell Complexes

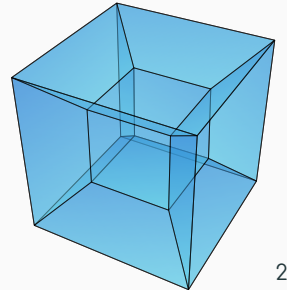
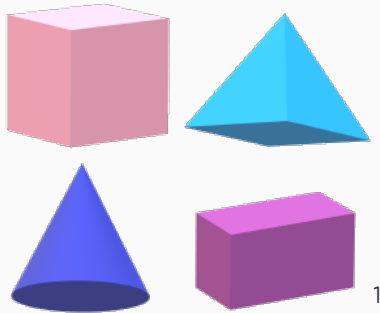








Usually represented as Matrices \Rightarrow Linear Algebra

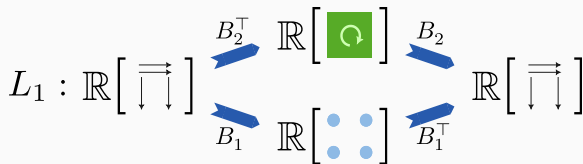


¹CC-BY 4.0 — https://commons.wikimedia.org/wiki/File:3D_Shapes.png

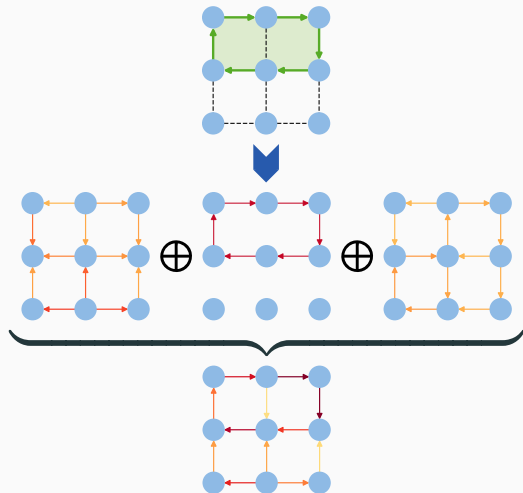
²CC-BY-SA 3.0 — <https://commons.wikimedia.org/wiki/File:Hypercube.svg>

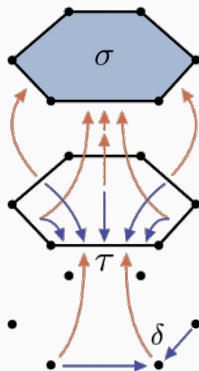
Applications & Methods

Hodge Laplacian: $L_k = B_k^\top B_k + B_{k+1} B_{k+1}^\top$



- Generalization of Graph Signal Processing
- Hodge Decomposition





From: Bodnar et al. *Weisfeiler and Lehman Go Cellular: CW Networks*. NeurIPS 2021. Image (c) 2021 The CWN Project Authors (MIT License <https://github.com/twitter-research/cwn/blob/main/LICENSE>)

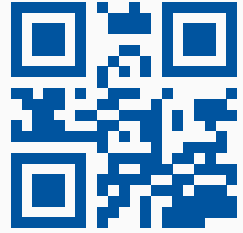
Random Abstract Cell Complexes

Josef Hoppe & Michael T. Schaub (hoppe.schaub@cs.rwth-aachen.de)

The poster is divided into several sections:

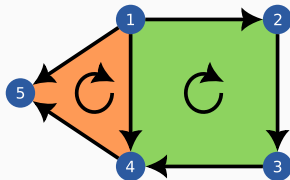
- Model:** A flowchart showing the process from a graph to a cell complex, involving operations like '1. contract', '2. collapse', and '3. contract'.
- Applications:** A list of applications including 'Geometric Interpretation', 'Central: Boundary Map', 'Topological Signal Processing', and 'Topological Deep Learning'.
- Sampling 2-cells (Simple Cycles):** A diagram showing the sampling process from a graph to a cell complex.
- Problem Complexity:** A graph showing the complexity of the problem.
- Calculating the Occurrence Probability:** A diagram showing the calculation of occurrence probability.
- Efficient Approximation:** A diagram showing an efficient approximation method.
- Strengths:** A list of strengths including 'Efficient Approximation', 'Geometric Interpretation', and 'Topological Signal Processing'.
- Weaknesses:** A list of weaknesses including 'Combinatorial Complexity' and 'Inference of Cells: No Ground Truth'.
- Background Cell Complexes:** A diagram showing the background cell complexes.
- Secondary Use Case: Counting Cycles:** A diagram showing the secondary use case of counting cycles.
- References:** A list of references.
- Funding Declaration:** A section for funding declaration.

- Abstract Cell Complexes add Structure to Graphs
 - Geometric Interpretation
 - Central: Boundary Map
- Applications
 - Topological Signal Processing
 - Topological Deep Learning
- Challenges
 - Inference of Cells: No Ground Truth
 - Combinatorial Complexity



hoppe.io
For
Introduction
Paper: E-mail
or LinkedIn

Boundary Maps: Example



$$B_1 = \begin{matrix} & 1 \rightarrow 2 & 1 \rightarrow 4 & 1 \rightarrow 5 & 2 \rightarrow 3 & 3 \rightarrow 4 & 4 \rightarrow 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

$$B_2 = \begin{matrix} & \triangleleft & \square \\ \begin{matrix} 1 \rightarrow 2 \\ 1 \rightarrow 4 \\ 1 \rightarrow 5 \\ 2 \rightarrow 3 \\ 3 \rightarrow 4 \\ 4 \rightarrow 5 \end{matrix} & \begin{pmatrix} 0 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \end{matrix}$$