Introduction to Abstract Cell Complexes and Topological Signal Processing

A Network Science Perspective

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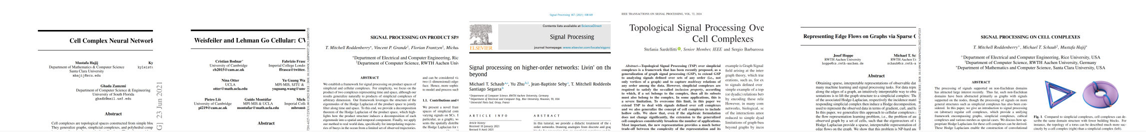
November 29, 2024

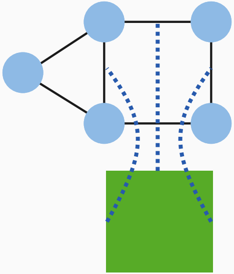
Learning on Graphs — Paris Meetup

Computational Network Science

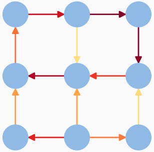
RWTH AACHEN UNIVERSITY

Motivation





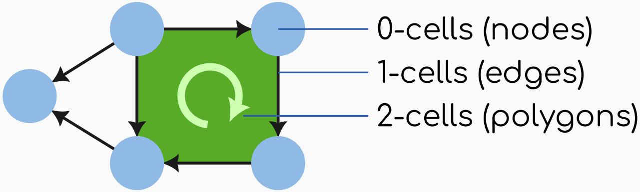
- Higher-Order Structure increases Expressiveness
- CCs enable Signal Processing over Edge Data



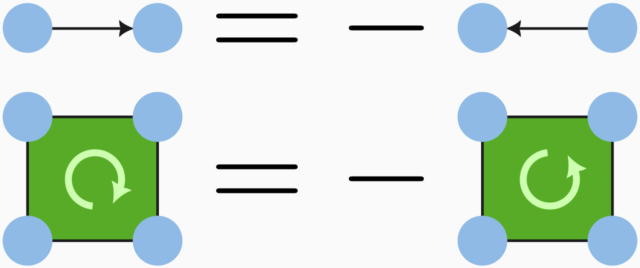
Today: Provide Intuitive Understanding

Abstract Cell Complexes

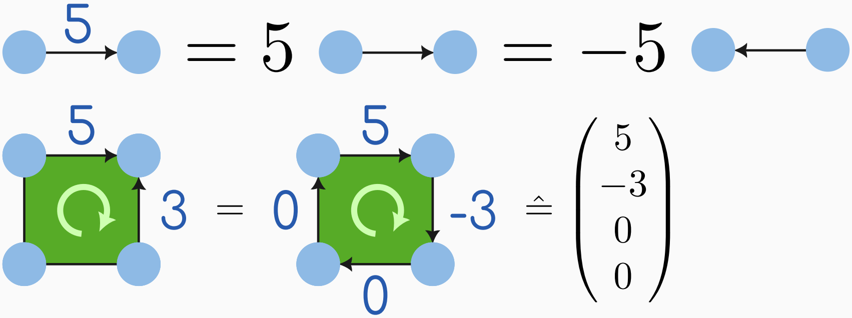
Intuition: 2-dim CCs

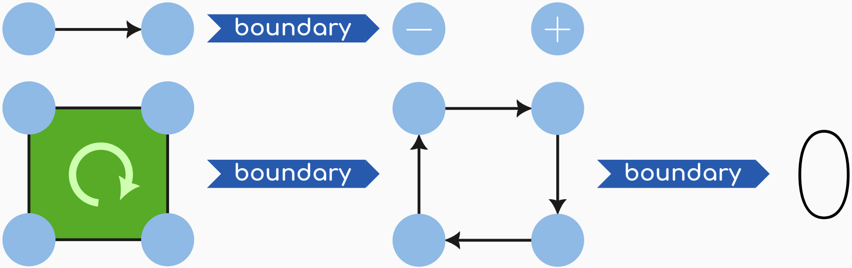


Orientation



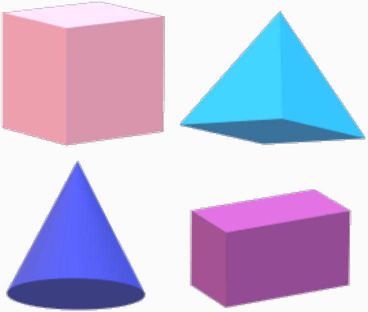
Data on Cells: (Co-) Chains



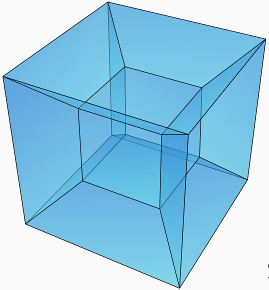


Usually represented as Matrices ⇒ Linear Algebra

Higher-Dimensional Cells



1

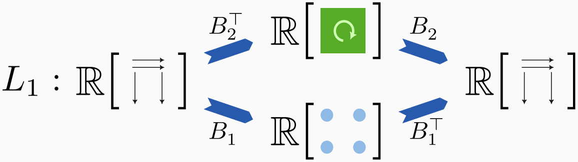


2

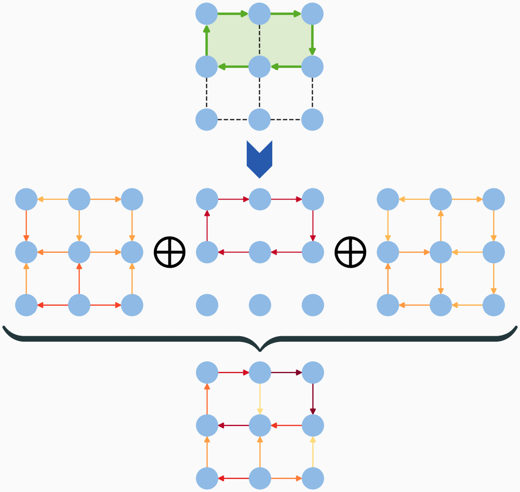
Applications & Methods

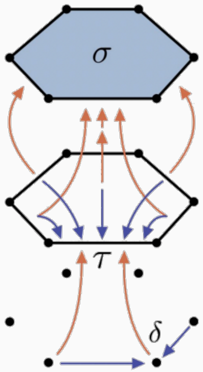
Topological Signal Processing

Hodge Laplacian: $L_k = B_k^\top B_k + B_{k+1} B_{k+1}^\top$



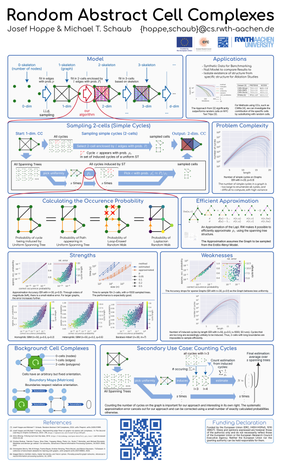
- Generalization of Graph Signal Processing
- Hodge Decomposition





From: Bodnar et al. *Weisfeiler and Lehman Go Cellular: CW Networks*. NeuRIPS 2021. Image (c) 2021 The CWN Project Authors (MIT License `https://github.com/twitter-research/cwn/blob/main/LICENSE`)

Conclusion



- Abstract Cell Complexes add Structure to Graphs
  - Geometric Interpretation
  - Central: Boundary Map
- Applications
  - Topological Signal Processing
  - Topological Deep Learning
- Challenges
  - Inference of Cells: No Ground Truth
  - Combinatorial Complexity

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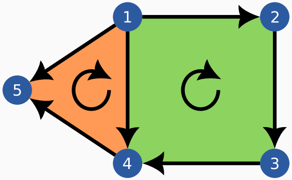
For Introduction Paper: E-mail or LinkedIn

Boundary Maps: Example



$$
B_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c} 1\to 2 \quad 1\to 4 \quad 1\to 5 \quad 2\to 3 \quad 3\to 4 \quad 4\to 5 \\ \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array}
$$

$$
B_2 = \begin{array}{c} 1\to 2 \\ 1\to 4 \\ 1\to 5 \\ 2\to 3 \\ 3\to 4 \\ 4\to 5 \end{array}
\begin{pmatrix} 0 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}
$$